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## RESISTANCE TO MOTION OF BODIES

## IN A FLUIDIZED BED

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The force resisting the motion of bodies of revolution in a horizontal direction in a fluidized bed is measured. A generalization of the experimental data is used to obtain a relationship estimating the resistive forces arising in a fluidized bed for flow around obstacles.

In industrial apparatuses, fluidized beds are used to wash around a variety of bodies (heat-exchanger surfaces, pipes for distributing reagents, immobile sediments, sensors, and so on). In some kiln constructions components being heat treated are moved through a fluidized bed. In all cases, in order to calculate the strength of the mountings, supports, and other structural elements, one requires information on the forces arising from the flow around bodies of a fluidized bed. The data published in the literature on this topic are very limited and relate to the average vertical forces acting on fixed model bodies immersed in the fluidized bed [1-3]. At the same time, as correctly noted in [1, 3], the instantaneous forces arising from the flow around a body of a dispersed material that is transported upward in the wake of a bubble are much greater than the average forces, in some cases by more than an order of magnitude. In order to be able to calculate the instantaneous forces in a fluidized bed, we need to know, besides the velocity of ascent of the bubbles, the dependence of the force on the velocity of motion the surrounding medium, on the characteristics of the medium, and on the size and shape of the body around which the medium is flowing. The present paper reports an experimental study of this question.

The experiments were performed in a column of cross section $275 \times 70 \mathrm{~mm}$ which had a gas-distributing grid made from a sheet of felt of thickness 6 mm . The dispersed material was three fractions of quartz glass ( $d=0.15,0.23$, and $0.63 \mathrm{~mm} ; u_{0}=4,6$, and $32 \mathrm{~cm} / \mathrm{sec}$ ) and one fraction of silica gel ( $d=0.19 \mathrm{~mm} ; u_{0}=$ $2 \mathrm{~cm} / \mathrm{sec}$ ). Fluidization was by means of air at room temperature for N from 1 to 5 . The initial height to which the column was filled was 29 cm . A dynamometer sensor was moved backward and forward along the horizontal (more accurately, along an arc of radius 45 cm ) by means of a special crank/connecting rod


Fig. 1. Dependence of force of resistance $F(N)$ for a sphere on its velocity of motion $u(\mathrm{~m} / \mathrm{sec})$ in a fluidized bed of quartz sand $\mathrm{d}=0.23 \mathrm{~mm}$ : points $1,2,3$, and 4 correspond, respectively, to fluidization numbers $1.2,2,3$, and 5 for a height above the grid of 125 mm . Points 5, 6, 7, and 8 correspond to fluidization numbers $1.2,2,3$, and 5 for a height above the grid of 225 mm .

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Fig. 2. Dependence of the quantity $F / \rho\left(\mathbb{N} \cdot \mathrm{m}^{3} / \mathrm{kg}\right)$ on velocity of motion $u$ for various materials: 1) quartz sand ( $\left.\rho=2650 \mathrm{~kg} / \mathrm{m}^{3}\right) ; 2$ ) silica gel $\left(\rho=1100 \mathrm{~kg} / \mathrm{m}^{3}\right)$.
mechanism. The amplitude of the displacement of the sensor was around 20 cm . Its velocity of motion was close to sinusoidal. It could be varied from 0.1 to $2 \mathrm{~m} / \mathrm{sec}$ to within $1 \%$ by varying the number of revolutions of the crank/connecting rod mechanism.

The dynamometer-sensor consists of an elastically restrained phosphor-bronze plate of dimensions $14 \times 6 \mathrm{~mm}$ with a stiff needle of diameter 0.5 mm and length 28 mm fixed to the free end. The sensor was displaced in a direction perpendicular to the plane of the elastic plate. Film-type tensometer sensors with a base of 5 mm and a resistance of $100 \Omega$ were stuck to both sides of the plate. To prevent contact with solidphase particles the plate was fixed in a protective tubular jacket so that only the needle projected through a diametral slit on the end face of the tube. The body to be investigated (sphere, disk, spindle) was fixed on the end of the needle. In the experiments we used plastic spheres of diameter 5.5 and 7.9 mm , a disk of diameter 5.5 mm and thickness 0.2 mm made from brass, and a plastic spindle of diameter 5.5 mm and length 9 mm (apex angle of cone $53^{\circ}$ ). The spindle was moved in the direction of the major axis and the disk along the axis perpendicular to its plane.

In the experiments to measure the force of resistance associated with the displacement of the bodies in the fluidized bed the sensor was connected to a TA-5 tensometer amplifier, the output signal from which was recorded on the graph paper of an $\mathrm{N}-327$ pen recorder. A static calibration of the system was made before the measurements. A weight was suspended on the end of the needle, and the output signal was recorded. The force acting on the body was determined on the pen-recorder paper to within $5 \%$.

For a to-and-fro motion of the sensor the signal on the pen-recorder paper had a form close to sinusoidal. For each period the force of friction $F$ was determined at the moments of time the velocity of the sensor reached its maximum value. At these moments the sensor is moving without acceleration and the attached mass consequently equals zero. In each mode the value of $F$ was determined as the average of all the maximum values of the force in a time of around 2 min .

The first set of experiments was made with a sphere of diameter 5.5 mm which was moved in a fluidized bed of quartz sand $d=0.23 \mathrm{~mm}$ at two levels: at heights 125 and 225 mm above the gas-distributing grid. The results of the experiments are presented in Fig. 1. The experimental points are grouped around a straight line with a mean-square scatter of around $10 \%$. It can be seen from these results that neither the fluidization number (which varied from 1 to 5) nor the distance from the gas-distributing grid has any significant effect on the force of friction; it depends linearly only on $u$. These results indicate, in particular, that the rheological characteristics of the emulsion phase of the fluidized bed, which in our experiments amounted to more than $80 \%$ of the volume of the bed, are stable in height and independent of the fluidization number.

The next set of experiments were performed with materials of appreciably different densities (sand; silica gel). Figure 2 shows that in the adopted coordinates the experimental points are generalized by a single straight line. This dependence indicates that the resistive force opposing the motion of a body in a fluidized bed is a linear function of the density of the dispersed material.

In order to generalize all the experimental data obtained by us (including also experiments with bodies of various shapes) recourse was made to dimensional analysis allowing for the results in Figs. 1 and 2 and the coordinates Eu and Fr adopted. The results of this treatment of the experimental data are shown in Fig. 3. They can be approximated by the relationship


Fig. 3. Dependence of Eu on Fr: 1, 2, 3) sphere $D=5.5 \mathrm{~mm}$, sand d equal to $0.6,0.15$, and 0.23 mm ; 4) sphere $D=5.5 \mathrm{~mm}$, silica gel $d=0.19 \mathrm{~mm} ; 5,6,7$ ) sand $d=0.23 \mathrm{~mm}$ and, respectively, a sphere $\mathrm{D}=7.9 \mathrm{~mm}$, a spindle $\mathrm{D}=5.5 \mathrm{~mm}$, and a disk $\mathrm{D}=5.5 \mathrm{~mm}$.

$$
\begin{equation*}
\mathrm{Eu}=p \mathrm{Fr}^{q} \tag{1}
\end{equation*}
$$

The values of the quantities $p$ and $q$ were determined by the method of least squares. Furthermore, we esti-mated the confidence interval [4] for $q$. It turned out that the value of $q=0.54 \pm 0.05$ with a probability of 0.95 . We took $q=0.5$, which corresponds to $p=49$. The mean-square scatter of the experimental points about the obtained relationship amounts to around $20 \%$.

Expression (1) can be written in dimensional form:

$$
\begin{equation*}
F=49 \rho u D^{2} \sqrt{g d} \tag{2}
\end{equation*}
$$

The fact that the force of resistance depends linearly on the velocity, is proportional to the area of the mid-section (i.e., to $\mathrm{D}^{2}$ ), and is independent of the shape of the body (in Fig. 3 all the bodies used in our experiments form a common field of points) reflects the rheological peculiarities of the emulsion phase of a fluidized bed.

Like an ideal free-flowing medium [5], the emulsion phase can be regarded as behaving as a Newtonian fluid only up to a definite level of compressive stresses. Above this level the medium loses mobility and behaves as a rigid body. One would expect on the basis of these ideas a zone of immobile particles to be formed in front of a moving body, where the compressive stresses are greatest, this zone moving along with the body. In this case the size of the surface on which the forces of friction act depends not on the shape of the body, but on the properties of the dispersed medium (its limiting compressive stress) and on the area of the mid-section of the body. At the same time, forces of friction on the rear surfaces of the body can be neglected, as in this zone the particles are spaced from the body. The formation of a cavity behind a moving body was observed visually in the motion of a sphere near the top surface of the fluidized bed. Fountains appeared in this cavity, while no fountaining was observed in front of the sphere.

The possible existence of a zone with immobile particles in front of a body moving in a fluidized bed is mentioned in [6] in connection with the analysis of data on the local coefficient of heat exchange.

It is useful to compare the above results with the data in [3], in which a study was made of the timeaveraged vertical force acting on horizontal disks of diameter from 20 to 80 mm immersed in a fluidized bed. The correlation obtained in [3] implies that

$$
\begin{equation*}
F_{\mathrm{av}} \sim \rho D^{1.48} \tag{3}
\end{equation*}
$$

The force is proportional to the density of the dispersed material and to the diameter of the body raised to a power greater than unity. This result serves as an indirect confirmation of relationship (2), from which it follows that the force of resistance varies in proportion to the density of the material and is nonlinearly dependent on the size of the body. The difference both in the experimental methods and in the aims of the investigations makes it difficult to compare the present results with the data in [3] on the effect of other quantities
on the force of resistance. In particular, it is not possible to obtain from [3] a direct connection between the force of resistance and the velocity of the dispersed medium relative to the body.

## NOTATION

$\mathrm{d}, \mathrm{D}$, diameter of the particles of the dispersed medium and the diameter of the midsection of the body, respectively; $u$, velocity of displacement of body; $u_{0}, u_{1}$, velocities of the beginning of fluidization and filtration, respectively; $N=u / u_{0}$, fluidization number; $\rho$, density of particles of solid phase; $g$, acceleration due to gravity; $F$, force-resisting motion of body in fluidized bed; $F_{a v}$, average vertical force acting upon a body in a fluidized bed; $\mathrm{Eu}=\mathrm{F} / \rho \mathrm{u}^{2} \mathrm{D}^{2}$, Euler number; $\mathrm{Fr}=\mathrm{gd} / \mathrm{u}^{2}$, Froude number.

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## MASS EXCHANGE BETWEEN A SOLID SPHERE AND

## A LIQUID IN CROSSED ELECTRIC AND MAGNETIC FIELDS

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The results of an experimental investigation of the effect of the magnitude and direction of the ponderomotive force $[j \times B$ ] on the coefficient of mass transfer from an electrically nonconducting sphere to a liquid are described.

The method considered here consists essentially in using crossed electric and magnetic fields to change the effective density of a current-carrying liquid by means of interaction between an external magnetic field and the liquid [1]. As is known, under the action of a magnetic field, a current-carrying liquid becomes effectively heavier or lighter, depending on the direction of the field. For certain geometric characteristics of the system (dissolution apparatus design), this may result in liquid motion. External mass exchange between a solid particle and the liquid is accelerated under these conditions due to forced convection. Moreover, intensification of mass exchange is connected with the translational motion of the liquid. Theoretical investigations [2] show that circulation flow, which promotes mass exchange, arises near curved surfaces of an object immersed in a current-carrying liquid.

Our aim was to investigate experimentally the intensification of external mass exchange in crossed electric and magnetic fields on the example of the dissolution of pressed spherical specimens of $\mathrm{KNO}_{3}$ salt in a $10 \%$ solution of $\mathrm{KNO}_{3}$.

The experiments were performed by means of a device (Fig. 1) based on an electromagnet [3], where a transparent-plastic vessel ( $200 \times 200 \times 25 \mathrm{~mm}$ ) containing the operating solution is placed between the pole pieces of the electromagnet. Stainless-steel electrodes $(40 \times 18 \times 1 \mathrm{~mm})$, spaced at 42 mm , are fastened in the upper part of the vessel (Fig. 2). With the superposition of a crossed field, the liquid between the electrodes either drops or rises, depending on the direction of the current. As a result, the solution in the vessel

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